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Problem set 2: Seminars – ECON 4335 Economics of Banking
March 2 & 4 – 2010 (week 9)

Problem 1. Consider a profit-maximizing bank, as in 3.1.3 in F&R, taking all interest rates (r_D, r_L, r) as given. How will, the bank's positions (D, L, m) be affected by:

- A higher capital requirement α
- A higher rate of interest in the interbank market

A guideline to solution:
We have a large number of banks, taking prices which are the various rates of interest as given (exogenous).
Let r_L be the given rate of interest on loans, r_D on deposits, and r the one in the interbank market.

For ease of exposition we assume that banks are identical, with profits

$$\pi = r_L L + rm - r_D D - C(D, L)$$

where m is the net position of a representative bank in the interbank market, as given by – cf. the balance sheet:

$$m = D - L - K = D - L - \alpha D = (1-\alpha)D - L$$

Here $\alpha \in (0,1)$ is the fraction of deposits held in liquid reserves (K) – a policy instrument. This reserve will have a cost, because it limits the funds provided for long-term investment. (If $m < 0$, then the bank under consideration has debt to other banks, borrowing from other banks has a cost; if $m > 0$, the bank has claims on other banks; lending to other banks provides revenues.) Dropping the sub- and superscripts, we then have:

$$\pi = r_L L + r\left[(1-\alpha)D - L\right] - r_D D - C(D, L) = (r_L - r)L - \left[r_D - r(1-\alpha)\right]D - C(D, L)$$

The bank from the point of view of a lender, the interbank rate of interest serves as an opportunity cost to loans. As a borrower, the **effective cost** or effective rate of interest on deposits, is $r_D - (1-\alpha)r$. A unit deposit will be charged or cost the bank r_D. A fraction α of a unit deposit has to be kept in none-interest bearing reserves, but the remaining fraction, $1-\alpha$, can be lent to other banks in the interbank market at the rate of interest r. The effective cost per unit deposit to the bank is therefore $r_D - (1-\alpha)r$. In a similar way; $r_L - r$ is the effective revenue per unit lent outside the interbank market.

Given that we have an interior profit-maximizing pair (D*, L*); it must obey:

$$\frac{\partial \pi}{\partial D} = \left[r(1-\alpha) - r_D\right] - C_D = 0$$
$$\frac{\partial \pi}{\partial L} = (r_L - r) - C_L = 0$$

For each output, a profit-maximizing bank being price-taker in all segments, will adjust quantities, i.e. deposits, resp. loans, so that the marginal cost of each output is equal to the intermediation margins or "output" price.

The cost function is assumed to be strictly twice continuously differentiable, strictly increasing and convex, with $C_j > 0$, $C_{jj} \geq 0$ for $j = L, D$, and $\Lambda := C_{DD}C_{LL} - (C_{LD})^2 > 0$. (Note that in the book one uses $\delta := -\Lambda$.) The sign of the second-order cross derivative $C_{DL} = C_{LD}$ tells us whether we have economies of scope $C_{LD} := \dfrac{\partial^2 C}{\partial L \partial D} < 0$ or diseconomies of scope $C_{LD} > 0$. We have economies of scope if the marginal cost in one activity is reduced as the scale of the other activity is increased; and diseconomies of scope in the opposite case. In the book (pp. 72-74) it is shown that:

- L will be increasing when r_L increases
- D will be decreasing when r_D increases
- How L is affected by a higher r_D, or how D is affected by a higher rate of interest on loans, r_L, will depend on $C_{LD} := \dfrac{\partial^2 C}{\partial L \partial D}$. The intuition is: Let $x := \left[r(1-\alpha) - r_D\right]$ and $y := r_L - r$. Then, we have the supply function $L(x, y)$ for loans and demand for deposits as given by $D(x, y)$. For fixed *y*, we get:

$$\left[r(1-\alpha) - r_D\right] - C_D = 0 \Rightarrow 1 = C_{DD}\frac{\partial D}{\partial x} + C_{DL}\frac{\partial L}{\partial x}$$
$$(r_L - r) - C_L = 0 \Rightarrow 0 = C_{LD}\frac{\partial D}{\partial x} + C_{LL}\frac{\partial L}{\partial x}$$

We can solve for $D_x := \dfrac{\partial D}{\partial x}, \, and \, L_x := \dfrac{\partial L}{\partial x}$. On writing the system of equations using matrix notation, we get:

$\begin{bmatrix} C_{DD} & C_{DL} \\ C_{DL} & C_{LL} \end{bmatrix} \cdot \begin{bmatrix} D_x \\ L_x \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ from which we can find directly the own-price effect on the demand for deposits, by using Cramer's rule; $D_x = \dfrac{C_{LL}}{\Lambda} > 0$ and the cross-price effect on the supply of loans $L_x = \dfrac{-C_{LD}}{\Lambda} > 0$ if $C_{LD} < 0$, and negative if $C_{LD} > 0$. L_x shows the cross-price effect; i.e. the impact on the amount of loans from a change in the effective cost of deposits.

The first case, with $C_{LD} = \frac{\partial}{\partial L} C_D < 0$, refers to **economies of scope** – marginal cost for one output is reduced ("shifts downwards") the more is produced of the other output. Suppose r_D goes down (x goes up), then demand for deposits increases. The supply of loans will, with economies of scope be positively affected, as C_L is reduced. Economies of scope might support universal banking.

The second case, $C_{LD} = \frac{\partial}{\partial L} C_D > 0$ is called "diseconomies of scope" – marginal cost in one activity is increasing ("shifts upward") in the other output. Diseconomies of scope may support specialized banks.

If at last, there is no dependence; $C_{LD} = 0$, there is no cost interaction between the two activities. (There is neither a gain nor a cost of undertaking both activities.)

We also have: $\begin{cases} 0 = C_{DD} D_y + C_{DL} L_y \\ 1 = C_{LD} D_y + C_{DL} L_y \end{cases} \Rightarrow \begin{bmatrix} C_{DD} & C_{DL} \\ C_{DL} & C_{LL} \end{bmatrix} \cdot \begin{bmatrix} D_y \\ L_y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ with $\begin{cases} D_y = \dfrac{-C_{DL}}{\Lambda} \\ L_y = \dfrac{C_{DD}}{\Lambda} > 0 \end{cases}$

with similar signs as above: L will be increasing in $y := r_L - r$ whereas deposits will increase under economies of scale and decline with diseconomies of scale when y increases.

Going back to the optimality conditions:

$$[r(1-\alpha) - r_D] - C_D = 0 \Leftrightarrow x - C_D(D(x,y), L(x,y)) = 0$$
$$(r_L - r) - C_L = 0 \Leftrightarrow y - C_L(D(x,y), L(x,y)) = 0$$

A higher capital requirement:

Rather than using x, y as above, we have $D(\alpha, r_L, r_D, r)$ and $L(\alpha, r_L, r_D, r)$, and we want to find the sign on $\frac{\partial D}{\partial \alpha}$ and $\frac{\partial L}{\partial \alpha}$, from the system:

$$-r - C_{DD} \frac{\partial D}{\partial \alpha} - C_{DL} \frac{\partial L}{\partial \alpha} = 0 =$$
$$0 - C_{LD} \frac{\partial D}{\partial \alpha} - C_{LL} \frac{\partial D}{\partial \alpha} = 0$$

When $\frac{\partial D}{\partial \alpha} := D_\alpha$ and $\frac{\partial L}{\partial \alpha} := L_\alpha$, we get:

$$\begin{bmatrix} C_{DD} & C_{DL} \\ C_{DL} & C_{LL} \end{bmatrix} \cdot \begin{bmatrix} D_x \\ L_x \end{bmatrix} = \begin{bmatrix} -r \\ 0 \end{bmatrix}; \text{ hence } \begin{cases} D_\alpha = \dfrac{\begin{vmatrix} -r & C_{DL} \\ 0 & C_{LL} \end{vmatrix}}{\Lambda} = \dfrac{-rC_{LL}}{\Lambda} < 0 \\ L_\alpha = \dfrac{\begin{vmatrix} C_{DD} & -r \\ C_{DL} & 0 \end{vmatrix}}{\Lambda} = \dfrac{rC_{LD}}{\Lambda} \end{cases}$$

When α increases, it will become more expensive to operate deposits; hence the demand for deposits will go down. How supply of loans will be affected follows from whether the bank's cost function exhibits economies or diseconomies of scope. One should perhaps expect that a higher capital requirement was used as an instrument to get the bank to contract its activities. This is the case only if the cost function exhibits economies of scope. If diseconomies of scope, $C_{LD} > 0$, the supply of loans will increase as reduced demand for deposits will lower the marginal management cost of lending. What about the interbank position?
From $m = (1-\alpha)D - L = (1-\alpha)D(\alpha, r_L, r_D, r) - L(\alpha, r_L, r_D, r)$, we find

$$\frac{\partial m}{\partial \alpha} = -D + (1-\alpha)D_\alpha - L_\alpha = -D - r(1-\alpha)\frac{C_{LL}}{\Lambda} - r\frac{C_{LD}}{\Lambda}$$

The two first terms pushes towards lowering m and reinforced by the third term if there are diseconomies of scale.

A higher rate of interest rate in the interbank market:
The interbank rate of interest r increases, which will affect both prices, denoted x and y. On differentiating the system

$$\left[r(1-\alpha) - r_D\right] - C_D = 0$$
$$(r_L - r) - C_L = 0$$
$$m = (1-\alpha)D - L$$

with respect to the interbank rate of interest r, we get:

$$1 - \alpha - C_{DD}D_r - C_{DL}L_r = 0$$
$$-1 - C_{LD}D_r - C_{LL}L_r = 0$$
$$m_r = (1-\alpha)D_r - L_r$$

From the first two:

$$\begin{bmatrix} C_{DD} & C_{DL} \\ C_{LD} & C_{LL} \end{bmatrix} \cdot \begin{bmatrix} D_r \\ L_r \end{bmatrix} = \begin{bmatrix} 1-\alpha \\ -1 \end{bmatrix} \Rightarrow \begin{cases} D_r = \dfrac{1}{\Lambda}\begin{vmatrix} 1-\alpha & C_{DL} \\ -1 & C_{LL} \end{vmatrix} = \dfrac{\overbrace{C_{LL}(1-\alpha)}^{+} + C_{DL}}{\Lambda} \\ L_r = \dfrac{1}{\Lambda}\begin{vmatrix} C_{DD} & 1-\alpha \\ C_{LD} & -1 \end{vmatrix} = \dfrac{\overbrace{-C_{DD}}^{-} - C_{LD}(1-\alpha)}{\Lambda} \end{cases}$$

Demand for deposits will increase with the rate of interest in the interbank market if the cost function exhibits weak diseconomies of scope; $C_{DL} \geq 0$. In that case supply of loans will go down.

What about: $m_r = \dfrac{1}{\Lambda}\left[(1-\alpha)^2 C_{LL} + 2(1-\alpha)C_{DL} + C_{DD}\right]$ which is positive if $C_{DL} \geq 0$.

Problem 2. How is the competitive structure of equilibrium interest rates in 3.1.4 in F&R affected by:

- A higher capital requirement
- Open market operations through a change in B (the stock of government bonds)
- A more efficient banking industry, say by lowering the constant marginal costs $C_L = a$ and $C_D = b$ to ka and kb for some $k < 1$.

A tentative solution:

In a competitive banking equilibrium each bank is characterized by a supply function for loans; $L_j(\alpha, r_L, r_D, r)$ and a demand function for deposits $D_j(\alpha, r_L, r_D, r)$ for $j = 1, 2, \ldots, N$ where N is the number of banks – cf. the problem above.

Demand for loans is given by $I(r_L)$ which is a declining function of the rate of interest on loans. We can think of firms undertaking investment projects; the higher is the interest rate on loans, a smaller amount of investment projects will have a positive net present value; hence demand for loans decrease with r_L.

Supply of savings is given by a saving function $S(r_D)$; increasing in the rate of interest on deposits. We can think of households with financial surplus. A higher interest rate on deposits will, for savers, have opposing effects on their demand: A substitution effect making current consumption more expensive – leading to higher current saving, but also an opposing income effect, inducing less saving if consumption is not inferior. Here we assume the substitution effect dominates!

We also assume that holding treasury bills/ government bonds is a perfect substitute to banking deposits for households – hence, the rate of interest is the same for these.

We can put up the equilibrium conditions: Supply equals to (or above) demand for each type of activity.
With B as net supply of government bonds (if positive this will reflect that the government has a budgetary deficit – we can think of increasing its debt to private sector.) Then the equilibrium is characterized by:

(1) $I(r_L) = \sum_j L_j(\alpha, r_L, r_D, r)$ equilibrium in the market for loans

(2) $S(r_D) = B + \sum_j D_j(\alpha, r_L, r_D, r)$ equilibrium in the "savings" market

(3) $\sum_{j=1}^{N} m^j = 0$ aggregate positions in the interbank market vanish

We know that $m^j = D_j - L_j - K_j = D_j - L_j - \alpha D_j = (1-\alpha)D_j - L_j$. Using this relationship in (3) along with (1) and (2), along with constant marginal costs; $C_L = a$ and $C_D = b$, we get:

$$(3)^* \quad \sum_j L_j(\alpha, r_L, r_D, r) = (1-\alpha)\sum_j D_j(\alpha, r_L, r_D, r)$$

In equilibrium we must have "price equals to marginal cost" in any output; if price is above the constant marginal cost, a bank would increase output beyond any limit; if below; output would be driven down to zero. In any interior equilibrium, we must have: $[r(1-\alpha) - r_D] - b = 0 \quad and \quad (r_L - r) - a = 0$
From which we get:

$$r_L = r + a$$
$$r_D = r(1-\alpha) - b$$

Using these in $(3)^*$, we get one condition which determines a unique equilibrium interbank rate of interest:

$$\sum_j L_j(\alpha, r_L, r_D, r) =_{(1)} I(r_L) =_{(3)} (1-\alpha)\sum_j D_j(\alpha, r_L, r_D, r) =_{(2)} (1-\alpha)[S(r_D) - B]$$

Or when using our remaining conditions: $\begin{cases} r_L = r + a \\ r_D = r(1-\alpha) - b \end{cases}$

we get:

$$I(r+a)=(1-\alpha)\left[S(r(1-\alpha)-b)-B\right]\Leftrightarrow S(r(1-\alpha)-b)-\frac{I(r+a)}{1-\alpha}=B$$

$$\Rightarrow\quad r(B,\alpha;a,b)\Rightarrow\begin{cases} r_L=r+a=a+r(B,\alpha;a,b):=r_L(B,\alpha;a,b)\\ r_D=r(1-\alpha)-b=(1-\alpha)\cdot r(B,\alpha;a,b)-b:=r_D(B,\alpha,a,b)\end{cases}.$$

We get one equilibrium condition, when in equilibrium we must have $r_L=r+a$ and $r_D=r(1-\alpha)-b$, to determine a unique rate of interest in the interbank market as given by $r(B,\alpha;a,b)$. Given this equilibrium rate of interest, equilibrium deposit and equilibrium lending rate of interest follow.

1. A higher capital requirement:
On using: $I(r_L)=(1-\alpha)\left[S(r_D)-B\right]$ and the relationships between the loan and deposit rates and the interbank rate:

$$I'(r_L)\frac{\partial r}{\partial\alpha}=-[S(r_D)-B]+(1-\alpha)S'(r_D)\left[-r+(1-\alpha)\frac{\partial r}{\partial\alpha}\right]$$

We get: $\frac{\partial r}{\partial\alpha}\underbrace{\left[(1-\alpha)^2S'(r_D)-I'(r_L)\right]}_{+}=\left[S(r_D)-B\right]+r(1-\alpha)S'(r_D)>0\Rightarrow\frac{\partial r}{\partial\alpha}>0$

Then r_L will increase, but two opposing effects on the equilibrium deposit rate.

Why? When the capital requirement increases, the "first or initial"[1] effect is, for a fixed structure of interest rates, that $I>(1-\alpha)(S-B)$, exhibiting an excess demand for loans. But as α is being increased, with r kept unchanged, deposit rate of interest r_D will go down, lowering S. "Standard" line of reasoning outside equilibrium, tells us that the price of loans, r_L, will then be pushed upwards, lowering demand for loans, so as to return to the new equilibrium. Then r will increase as well, having a positive impact on r_D. Note that the deposit rate will be influenced by two opposing forces. In the new equilibrium, we'll have *a higher interbank rate of interest and a higher rate of interest on loans*, but the deposit rate of interest rate can go either way.

2. Open market operations:
Suppose that *B* is increased – government supply or sale of bonds is increasing – because of a budgetary deficit ($\Delta M+\Delta B=G-T>0$ for a closed economy.)
The first effect of a higher B is that $I>(1-\alpha)(S-B)$. To restore equilibrium, the demand for loans should go down and S should increase; but deposits will go down, as more bonds will be kept by households; accomplished by $r_L\uparrow$ and $r_D\uparrow$; hence $r\uparrow$. (Crowding out.)

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[1] Note that introducing a sequence of steps is outside the model – all changes take place instantaneously.

From: $D(r(1-\alpha)-b) = S(r(1-\alpha)-b) - B$, we get $(D'-S')(1-\alpha)\dfrac{\partial r}{\partial B} = -1$, where $D'(r_D) < 0$ and $S'(r_D) > 0$, we get $\dfrac{\partial r}{\partial B} > 0$.

3. A more efficient banking industry:
Suppose that $a \downarrow ka, b \downarrow kb$ for $k < 1$. In equilibrium we then have:

$$I(r+ka) = (1-\alpha)\left[S(r(1-\alpha)-kb) - B\right] \Rightarrow r(k).$$

Question: What is the impact of lowering k from the initial level $k = 1$?

We find: $I'(r_L)(r'(k)+a) = (1-\alpha)\left[S'(r_D)((1-\alpha)r'(k)-b)\right] = 0$ from which we find:

$r'(k)\cdot\underbrace{\left[(1-\alpha)^2 S'(r_D) - I'(r_L)\right]}_{+} = \underbrace{\left[(1-\alpha)bS'(r_D) + aI'(r_L)\right]}_{?}$. Hence, we cannot sign $r'(k)$ unambiguously, as $S' > 0 > I'$.[2]
(If demand for loans is inelastic ($I' \approx 0$) then r will go down as efficiency is improved. If $S' \approx 0$, then r will increase with efficiency. On the other hand if $I' \to -\infty$ (elastic demand for loans), we have $r'(k) \to -a$, as $r_L = r + a$ is close to be constant. If $S' \to \infty$, then $r'(k) \to \dfrac{b}{1-\alpha}$, as in this case $r_D = (1-\alpha)r - b$ is close to be constant.)

One might benefit from considering optimal bank behavior from Problem 1 above, when introducing an efficiency parameter β in the cost function; $C(D, L;\beta)$. A higher value of β, will reduce both marginal costs; i.e. $\dfrac{\partial}{\partial\beta}C_D = C_{D\beta} < 0$ and $\dfrac{\partial}{\partial\beta}C_L = C_{L\beta} < 0$. For given rates of interest, we have from Problem 1 (with positive and increasing marginal costs), when differentiating the first-order conditions with respect to β, and using that $L = L(\beta)$ and $D = D(\beta)$, for fixed values of (r, r_L, r_D, α):

$$0 - C_{DD}D'(\beta) - C_{DL}L'(\beta) - C_{D\beta} = 0$$
$$0 - C_{LD}D'(\beta) - C_{LL}L'(\beta) - C_{L\beta} = 0$$

which can be written as $\begin{bmatrix} C_{DD} & C_{DL} \\ C_{DL} & C_{LL} \end{bmatrix}\begin{bmatrix} D'(\beta) \\ L'(\beta) \end{bmatrix} = \begin{bmatrix} -C_{D\beta} \\ -C_{L\beta} \end{bmatrix}$ with solution:

[2] The conclusion drawn at the seminar, I think was incomplete!

$$D'(\beta) = \frac{1}{\Lambda}\begin{vmatrix} -C_{D\beta} & C_{DL} \\ -C_{L\beta} & C_{LL} \end{vmatrix} = \frac{\overbrace{-C_{LL}C_{D\beta}}^{+} + C_{DL}C_{L\beta}}{\Lambda}$$

$$L'(\beta) = \frac{1}{\Lambda}\begin{vmatrix} C_{DD} & -C_{D\beta} \\ C_{DL} & -C_{L\beta} \end{vmatrix} = \frac{\overbrace{-C_{DD}C_{L\beta}}^{+} + C_{DL}C_{D\beta}}{\Lambda}$$

Observe: If there are (weak) economies of scope, $C_{DL} \leq 0$ then both demand for deposits and supply of loans for a price-taking bank will be positively affected by higher efficiency. (Then marginal cost is reduced; for given prices, individual outputs will go up.) If, however, the cost function exhibits diseconomies of scope the positive effects need to be adjusted downwards. Suppose therefore that both D and L will increase with efficiency; hence we have $D(\beta)$ and $L(\beta)$ both being increasing functions of the efficiency parameter β.

To see how the structure of equilibrium interest rates might be affected, we can see how increased efficiency on the individual level can be translated into an aggregated impact on the equilibrium conditions. To get firms to increase their demands for loans, r_L should be reduced, because we have $I(r_L) = \sum_j L_j(\alpha, r_L, r_D, r, \beta)$. When efficiency ($\beta$) increases the RHS of this equilibrium condition will increase. To reach the new equilibrium, r_L has to go down. For a given economic policy, with B fixed, an increased demand for deposits due to improved efficiency, will increase the RHS of $S(r_D) = B + \sum_j D_j(\alpha, r_L, r_D, r, \beta)$; hence for this to hold as an equality; the deposit rate of interest; r_D has to increase. Increased overall efficiency will therefore push r_L down and r_D up. How these changes are translated into the interbank market, will depend, among other things, on the magnitude of $\frac{S'}{-I'}$.